## 6.302 Prelab 1B

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1. By applying Black's law to the velocity control loop, we can see that

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$$\frac{\dot{\theta}}{\dot{\theta}_{in}} = \frac{-2G_c \frac{\dot{\theta}}{V_m}}{1 + 2G_c K_{tach} \frac{\dot{\theta}}{V_m}}$$

but since we know that

$$\frac{\dot{\theta}}{V_m} = \frac{K_T}{Js(R_m + sL_m) + K_TK_e}$$

we can say that

$$\frac{\dot{\theta}}{\dot{\theta}_{in}} = \frac{\frac{-2G_c K_T}{Js(R_m + sL_m) + K_T K_e}}{1 + \frac{2K_{tach}G_c K_T}{Js(R_m + sL_m) + K_T K_e}} = \frac{-2G_c K_T}{Js(R_m + sL_m) + K_T K_e + 2K_{tach} K_T G_c}$$

Assuming that  $L_m$  is small enough to ignore (i.e., the motor dynamics are dominated by the mechanical time constant rather than the electrical time constant), we can say that

$$\frac{\dot{\theta}}{\dot{\theta}_{in}} = \frac{-2G_C K_T}{JR_m s + K_T K_e + 2K_{tach} G_c K_T} = \left(\frac{-2G_c}{K_e + 2K_{tach} G_c}\right) \frac{1}{\tau s + 1}$$

where

$$\tau = \frac{R_m(J_m + J_f)}{K_T K_e + 2K_{tach} G_c K_T}$$

In order to make  $G_c = 25$  ms, we need to rearrange terms like so:

$$\tau K_T K_e + 2\tau K_{tach} G_c K_T = (J_m + J_f) R_m$$
$$G_c = \frac{R_m (J_m + J_f) - \tau K_T K_e}{2\tau K_T K_{tach}} = 0.641$$

2. Substituting experimental values, we can see that that

$$\tau = \frac{0.23}{0.4 + G_c}$$

This suggests that as the compensator gain increases, the open loop time constant will decrease.

3. For a unit step input, the system error is given by

$$E(s) = \left(\frac{-1}{s}\right) \frac{1}{1 + 2K_{tach}G_c\frac{\dot{\theta}}{V_m}} = \left(\frac{-1}{s}\right) \frac{1}{1 + \frac{2K_{tach}G_cK_T}{(J_m + J_f)sR_m + K_TK_e}}$$

The steady state error is then given by

$$\lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \frac{-1}{1 + \frac{2K_{tach}G_cK_T}{(J_m + J_f)sR_m + K_TK_e}} = \frac{-K_e}{K_e + 2K_{tach}G_c}$$

The DC closed loop gain of the velocity control system is given by

$$\lim_{s \to 0} \frac{\dot{\theta}}{\dot{\theta}_{in}} = \frac{-2G_c K_T}{K_T K_e + 2K_{tach} K_T G_c}$$

4. For a unit step input with a proportional plus integral compensator, the system error is given by

$$E(s) = \left(\frac{-1}{s}\right) \frac{1}{1 + \frac{2K_{tach}K_TG_c}{(J_m + J_f)R_m s + K_TK_e}} \frac{\tau s + 1}{\tau s}}{(J_m + J_f)R_m \tau s^2 + (K_TK_e + 2K_{tach}G_cK_T)\tau s + 2K_{tach}G_cK_T}}$$

The steady state error is then given by

$$\lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s) = 0$$

For a unit step input, the output is

$$\dot{\theta}(s) = \frac{-2K_T G_c}{(J_m + J_f)R_m \tau s^2 + (K_T K_e + 2K_{tach} G_c K_T) \tau s + 2K_{tach} G_c K_T} \left(\tau + \frac{1}{s}\right)$$

If we set  $G_c = \tau = 1$ , we can approximate the step response with

$$\dot{\theta}(s) = \left(\frac{1}{s}\right) \frac{-279(s+1)}{(s+3.86)(s+0.73)}$$

This response will be dominated by the lower frequency pole at s = 0.73 so the response will be approximately like the step response to a first order system.

5. For a proportional gain compensator in current drive, the closed loop transfer function is

$$\frac{\dot{\theta}}{\dot{\theta}_{in}} = \frac{-0.4G_c \frac{\dot{\theta}}{I_m}}{1 + 0.4G_c K_{tach} \frac{\dot{\theta}}{I_m}}$$

But since

$$\frac{\dot{\theta}}{I_m} = \frac{K_T}{(J_m + J_f)s}$$

we can say that

$$\frac{\dot{\theta}}{\dot{\theta}_{in}} = \frac{\left(\frac{-1}{K_{tach}}\right)}{\tau s + 1}$$

where

$$\tau = \frac{J_m + J_f}{0.4G_c K_T K_{tach}}$$

Hence

$$G_c = \frac{J_m + J_f}{0.4\tau K_T K_{tach}}$$

This suggests that in order to make  $\tau = 25$  ms, we need to make  $G_c = 5.19$ .

## 6. The system error when driven with a unit step input is given by

$$E(s) = \left(\frac{-1}{s}\right) \frac{1}{1 + 0.4K_{tach}G_c \frac{K_T}{(J_m + J_f)s}} = \left(\frac{-1}{s}\right) \frac{(J_m + J_f)s}{(J_m + J_f)s + 0.4G_c K_T K_{tach}}$$

The steady state error for a unit step input is then given by

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} s E(s) = 0$$

The closed loop DC gain is

$$\lim_{s \to 0} \frac{\dot{\theta}}{\dot{\theta}_{in}} = \frac{-1}{K_{tach}}$$

For a unit step input, the system output will be

$$\dot{\theta}(s) = \left(\frac{-1}{s}\right) \frac{\left(\frac{-1}{K_{tach}}\right)}{\tau s + 1}$$

where  $\tau$  is defined as in problem 5.